PROOF BY COUNTER EXAMPLES

Answer all of these questions. Remember to show your working out in all questions.

MAIN QUESTIONS

1. For all integers n, $n^2 + n + 41$ is prime

2. For all real numbers x, $x^2 > x$

3. For all integers n, $2n + 1$ is prime

4. For all real numbers a and b, $\sqrt{(a + b)} = \sqrt{a} + \sqrt{b}$

5. For all integers n, $n^2 - n + 11$ is prime

6. For all real numbers x, $|x| = x$

7. For all integers n, $n^3 - n$ is divisible by 3

8. For all real numbers x, $x^2 + 1 > 0$

9. For all integers n, $2^n - 1$ is prime

10. For all real numbers x, $\sin(x) < x$

11. For all integers n, $n^2 + n$ is even

12. For all real numbers x, $e^x > x + 1$

13. For all integers n, $n! + 1$ is prime

14. For all real numbers x, $x^3 > x^2$

15. For all integers n, $6n + 1$ is prime

16. For all real numbers x, $\ln(x^2) = 2\ln(x)$

17. For all integers n, $n^2 - 79n + 1601$ is prime

18. For all real numbers x, $\cos(x) \leq 1 - x^2/2$

19. For all integers n, $2^{(2^n)} + 1$ is prime

20. For all real numbers x, $(x + 1)^2 = x^2 + 1$

21. For all integers n, $n^4 + 4$ is prime

22.

23.

For all integers n, $n^2 + n + 17$ is prime

24.

For all real numbers x, $x/(x + 1) < 1$

25.

For all integers n, $10n + 1$ is prime

26.

For all real numbers x, $\sqrt{(x^2)} = x$

27.

For all integers n, $n^5 - n$ is divisible by 5

28.

For all real numbers x, $\sin(2x) = 2\sin(x)$

29.

For all integers n, $n^2 + 21n + 1$ is prime

30.

For all real numbers x, $e^{(-x)} = 1/e^x$

MASTER QUESTIONS

M1.

Every continuous function is differentiable

M2.

If a function is bounded, it must attain its maximum and minimum values

M3.

Every convergent sequence of real numbers is monotonic

M4.

If a series converges, then its terms must approach zero monotonically

M5.

Every subgroup of a cyclic group is cyclic

M6.

If two matrices commute, they must be diagonalisable

M7.

Every bounded sequence has a convergent subsequence

M8.

If a function has a local maximum at a point, then its derivative must be zero at that point

M9.

Every infinite set has the same cardinality

M10.

If a topological space is Hausdorff, then every sequence has at most one limit point