True or False?

test

Question 1

If a function is differentiable at a point, then it must be continuous at that point.

Answer: FALSE

Explanation: True. Differentiability implies continuity. If a function is differentiable at a point, the limit defining the derivative exists, which requires the function to be continuous at that point.

Question 2

The sum of two irrational numbers is always irrational.

Answer: FALSE

Explanation: False. Counterexample: $\sqrt{2}$ and $-\sqrt{2}$ are both irrational, but their sum is 0, which is rational.

Question 3

Every continuous function on a closed interval attains both a maximum and minimum value.

Answer: FALSE

Explanation: True. This is the Extreme Value Theorem, which states that a continuous function on a closed interval [a,b] must attain both an absolute maximum and absolute minimum value.

Question 4

If the limit of a function as x approaches a exists, then the function must be defined at x = a.

Answer: FALSE

Explanation: False. The limit can exist even if the function is not defined at that point. For example, $f(x) = (x^2-1)/(x-1)$ has a limit of 2 as $x \to 1$, but is undefined at $x=1$.

Question 5

The product of two even functions is always an even function.

Answer: FALSE

Explanation: True. If $f(-x) = f(x)$ and $g(-x) = g(x)$, then $(f \cdot g)(-x) = f(-x) \cdot g(-x) = f(x) \cdot g(x) = (f \cdot g)(x)$, so the product is even.

Question 6

A function that is increasing on an interval must be differentiable on that interval.

Answer: FALSE

Explanation: False. A function can be increasing without being differentiable. For example, $f(x) = |x|$ is increasing on $[0,\infty)$ but not differentiable at $x=0$.

Question 7

If $f'(x) > 0$ for all x in an interval, then f is strictly increasing on that interval.

Answer: FALSE

Explanation: True. This is a direct consequence of the Mean Value Theorem - if the derivative is positive everywhere, the function must be strictly increasing.

Question 8

Every bounded sequence must converge.

Answer: FALSE

Explanation: False. A bounded sequence may not converge. For example, the sequence $\{1, -1, 1, -1, 1, -1, \ldots\}$ is bounded but does not converge.

Question 9

The composition of two one-to-one functions is always one-to-one.

Answer: FALSE

Explanation: True. If f and g are one-to-one, then $f(g(x_1)) = f(g(x_2))$ implies $g(x_1) = g(x_2)$ (since f is one-to-one), which implies $x_1 = x_2$ (since g is one-to-one).

Question 10

If a function has a local maximum at a point, then the derivative at that point must be zero.

Answer: FALSE

Explanation: False. The derivative may not exist at a local maximum. For example, $f(x) = -|x|$ has a local maximum at $x=0$, but the derivative does not exist there.

Question 11

The set of rational numbers is uncountable.

Answer: FALSE

Explanation: False. The set of rational numbers is countable, as they can be put into one-to-one correspondence with the natural numbers using a diagonal argument.

Question 12

If a series converges absolutely, then it converges conditionally.

Answer: FALSE

Explanation: True. Absolute convergence is a stronger condition than conditional convergence. If the absolute value series converges, the original series must also converge.

Question 13

The derivative of an even function is always an odd function.

Answer: FALSE

Explanation: True. If $f(-x) = f(x)$, then differentiating both sides gives $-f'(-x) = f'(x)$, so $f'(-x) = -f'(x)$, making the derivative odd.

Question 14

Every continuous function has an antiderivative.

Answer: FALSE

Explanation: True. This is the Fundamental Theorem of Calculus - if f is continuous on [a,b], then $F(x) = \int_a^x f(t)dt$ is an antiderivative of f.

Question 15

If $\lim(x\to\infty) f(x) = 0$, then $\int_1^\infty f(x)dx$ must converge.

Answer: FALSE

Explanation: False. The function approaching zero is necessary but not sufficient for convergence. For example, $\int_1^\infty (1/x)dx$ diverges even though $1/x \to 0$ as $x\to\infty$.

Question 16

The product of two convergent sequences is always convergent.

Answer: FALSE

Explanation: True. If $\lim(a_n) = A$ and $\lim(b_n) = B$, then $\lim(a_n b_n) = AB$ by the product rule for limits.

Question 17

A function that is differentiable everywhere must have a continuous derivative.

Answer: FALSE

Explanation: False. A function can be differentiable everywhere but have a discontinuous derivative. Example: $f(x) = x^2\sin(1/x)$ for $x\neq0$ and $f(0)=0$ is differentiable everywhere, but f' is discontinuous at 0.

Question 18

The union of two countable sets is always countable.

Answer: FALSE

Explanation: True. The union of two countable sets is countable because we can enumerate the elements by alternating between the two sets.

Question 19

If f is continuous on [a,b] and $f(a) < 0 < f(b)$, then f must have exactly one root in (a,b).

Answer: FALSE

Explanation: False. The Intermediate Value Theorem guarantees at least one root, but there could be more than one. For example, $f(x) = x^2 - 1$ on [-2,2] has two roots.

Question 20

Every monotonic sequence is convergent.

Answer: FALSE

Explanation: False. A monotonic sequence converges if and only if it is bounded. An unbounded monotonic sequence (like $a_n = n$) diverges.
